Examinee's No.					

Department of Ocean, Policy, and Environment
Graduate School of Frontier Sciences
The University of Tokyo

Academic Year 2018 Entrance Examination for Master Course and Doctor Course

Logics and Mathematics

August 21, 2017

9:30～11:00（90min.）

Notice

1. Do not open this booklet until the start of the examination.
2. If you find missing pages, disorderly binding or unclear printing, raise your hand and consult a test proctor.
3. Write your examinee's number on the top of this sheet. Do not write your examinee's number or your name anywhere else.
4. There are 9 questions in total. Answer all of the 9 questions.
5. Write the outline of your solution below the question and put only the answer in the box.
6. Sheets for calculation are distributed separately.

Q 1

Define a complex variable $z = x + iy$ by independent real variables x and y, where i is the imaginary unit. Real and imaginary parts of a complex regular function $w = f(z)$ are expressed by $u(x, y)$ and $v(x, y)$, respectively. Find $v(x, y)$, when

$$u(x, y) = (x + 2)\{(x + 2)^2 - 3y^2\}.$$

Q 2

Let A be a 2x2 real matrix. Let $\begin{pmatrix} p \\ r \end{pmatrix}$ and $\begin{pmatrix} q \\ s \end{pmatrix}$ be non-zero vectors whose elements are real numbers. When $A\begin{pmatrix} p \\ r \end{pmatrix} = -\begin{pmatrix} q \\ s \end{pmatrix}$ and $A\begin{pmatrix} q \\ s \end{pmatrix} = \begin{pmatrix} p \\ r \end{pmatrix}$, answer the following questions.

(1) Prove that the matrix, $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$, has its inverse.

(2) Find A^2.

Q 3

At a supermarket, cart returns are placed at the east, central, and west gates. Shoppers can pick-up and return their carts at any gates. It has been observed that the following facts occur at the end of a day. 10% of the carts picked-up at the east gate are returned to the central gate, 10% to the west gate. 30% of the carts picked-up at the central gate are returned to the east gate, 20% to the west gate. 20% of the carts picked-up at the west gate are returned to the east gate, 10% to the central gate. Assuming there are 240 carts, how many carts should be placed at each gate so that the number of the carts at each gate does not change?

Q 4

The following matrix A can be expressed as the sum of a symmetric matrix P $(P = P^t)$ and a skew-symmetric matrix Q $(Q = -Q^t)$. Find P and Q.
Here, P^t and Q^t are the transposes of P and Q, respectively.

$$A = \begin{bmatrix} 3 & 2 & 3 \\ 0 & -2 & -6 \\ 7 & 0 & 7 \end{bmatrix}$$

$P =$

$Q =$

Q 5

Consider a regular triangle inscribed in a square. The side length of the square is 1.

(1) Draw the regular triangle in the square, when the triangle has the maximum area.

(2) Find the maximum area.

Ｑ６

O, A, B and C are vertices of a tetrahedron. Line segments OA, OB and OC are mutually orthogonal, and the lengths of OA, OB and OC are a, b and c, respectively. Find the length of the perpendicular line from the vertex O to the triangle ABC.

Ｑ７

A ship goes straight from the port A to the port B and it instantly returns to the port A by the same route. The ground speed of the ship is always constant in the no wind condition. On the other hand, when the wind blows from the port A to the port B, it is increased in a following wind and decreased in an opposing wind, by the same amount of speed. Compared with a round trip in the no wind condition, when the wind blows, a round trip of the ship spends

(1) more time.

(2) less time.

(3) the same amount of time.

Q 8

Train A arrives at a station randomly from the time T_0 to $T_0 + T$ minutes, and stops for α minutes in the station. Train B, independently of Train A, arrives at the station randomly during the same period, and stops for β minutes. Here, the probability distribution of the arrival time of the trains is uniform.

（１） Find the probability that Train A arrives at the station prior to Train B.

（２） Find the probability that both the trains are stopped in the station at the same time.

（３） Find the probability that Train A arrives at the station prior to Train B when both the trains are stopped in the station at the same time.

Q 9

$a, b,$ and c are natural numbers.

1) If $a + b + c = 20$, how many solutions exist for (a, b, c) ?

2) If $a + b + c \leq 20$, how many solutions exist for (a, b, c) ?

Examinee's No.					

Department of Ocean Technology, Policy, and Environment
Graduate School of Frontier Sciences
The University of Tokyo

Academic Year 2019 Entrance Examination for Master Course and Doctor Course

Logics and Mathematics

August 20, 2018

9:30～11:00（90min.）

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Q1

When real numbers a and b satisfy $a+2b=10$, find the minimum value of 2^a+16^b.

Q2

Obtain the values of the following definite integrals:

(1)

$$\int_0^{\pi} \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2 dx$$

(2)

$$\int_1^{e} (x + x^{-1})\, dx$$

Q3

Let A and h be positive real constants, and a and b are different positive real roots of the following equation;

$$A = x\tan(hx).$$

Suppose that functions $f_a(x)$ and $f_b(x)$ are defined as

$$f_a(x) = \cos(a(x-h)),$$
$$f_b(x) = \cos(b(x-h)).$$

Obtain the flowing definite integral;

$$I_{ab} = \int_0^h f_a(x) f_b(x) dx.$$

Q4

Answer the questions about the following matrix.

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

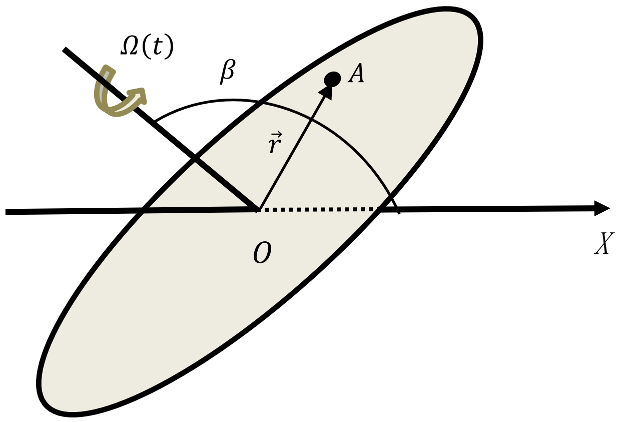
(1) Find the eigenvalues.

(2) Find the eigenvectors.

Q5

There is a disk rotating at an angular velocity $\Omega(t)$ around a rotation axis having an angle β with respect to the X axis fixed in the space, where t is the time. The rotational center of the disk is identical to the origin of a space-fixed coordinate system $(O - XYZ)$. A position vector $\vec{r}$ represents the position of a point A fixed on the disk

Obtain the acceleration vector at the point A by using the base vector $(\vec{\imath}, \vec{\jmath}, \vec{k})$ of the space-fixed coordinate system $(O - XYZ)$. Define the X and Y axes of the space-fixed coordinate system so that the answer can be written concisely.



Q6

Prove the following equation regarding △ABC.

$$a^2 = b^2 + c^2 - 2bc\cos\theta$$

Here, $a, b,$ and c are the lengths of the sides BC, CA, and AB, respectively, and θ is the angle of ∠CAB.

Q7

There are three cards. Each card has a different positive integer. Three people A, B and C randomly choose one card for each, and the number on the card is counted as a score for each. After repeating this game multiple times, the total scores, which A, B, and C obtained, were 10, 8, and 18, respectively. It is known that the card, which B chose, has the larger number than those of A and C in the second game. Answer the person who chose the card with the second largest number in the first game and explain its reason.

Q8

An integer N is described with a three-digit number as abc in quinary (base 5) notation, and also described with a four-digit number as $cdee$ in ternary (base 3) notation, where a and c are integers greater than or equal to 1, and b, d and e are integers greater than or equal to 0. Find all the combinations of the values, N, a, b, c, d, e, which satisfy the above conditions and describe them in decimal notation.

Q9

A drillship, which is a vessel designed for use in offshore drilling, has an ability of positioning by using her own thrusters based on the position information obtained from the position reference system. Suppose a drillship with six thrusters and one position reference system. The drillship loses the ability of positioning if more than or equal to two thrusters fail. Assuming that failure occurrence probabilities of each thruster and the position reference system are p and q, respectively, estimate the probability of the following cases.

(1) Probability that the drillship loses the ability of positioning due to thrusters' failures when the position reference system is working.

(2) Probability that the drillship keeps the ability of positioning.

Examinee's No.					

Department of Ocean Technology, Policy, and Environment
Graduate School of Frontier Sciences
The University of Tokyo
Academic Year 2020 Entrance Examination for Master Course and Doctor Course

Logics and Mathematics

August 19, 2019
9:30～11:00（90min.）

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Q1

Evaluate the following definite integral.

$$I = \int_{0}^{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 1}}$$

Q2

Variables x and y are defined as the following equations, where m is an integer and θ is an arbitrary real number.

$$\begin{cases} x(\theta) = \sum_{m=0}^{\infty} \frac{\theta^{2m}}{(2m)!} \\ y(\theta) = \sum_{m=0}^{\infty} \frac{\theta^{2m+1}}{(2m+1)!} \end{cases}$$

Note that $m! \equiv m \times (m-1) \times (m-2) \times \cdots \times 2 \times 1$, $0! = 1$, and $0^0 = 1$.
Answer the following questions.

1) Find $\frac{dx}{dy}$ using x and y.

2) Find the relationship between x and y.

Q3

Answer the following questions regarding

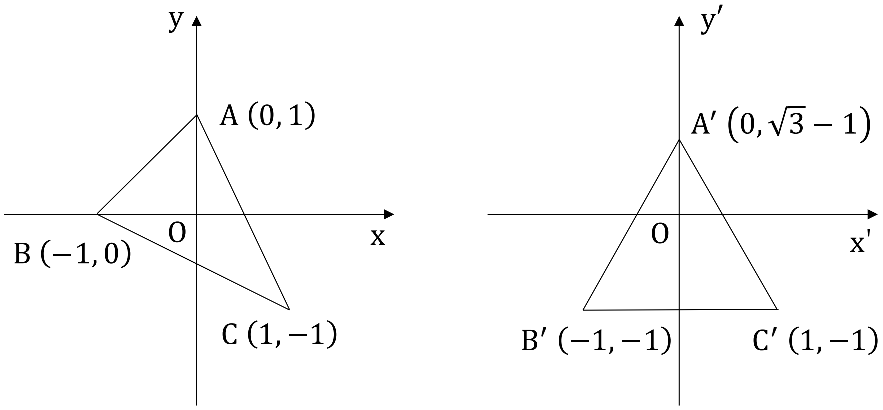
$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{pmatrix}.$$

1) Find A^{-1}.

2) Show that $|A^{-1}| = \frac{1}{|A|}$ is satisfied.

Q4

A triangle ABC on a plane Oxy was mapped to a triangle A'B'C' on a plane Ox'y'. Find the transformation that maps an arbitrary point $\vec{x}$ on a plane Oxy to a point $\vec{x'}$ on a plane Ox'y'.



Q5

Given the matrix $A = \begin{pmatrix} a & 1-a \\ 1-a & a \end{pmatrix}$, where a is a real number satisfying $0 < a < 1$, answer the following questions.

1) Find the eigenvalues and eigenvectors.

2) Find A^n where n is a natural number.

3) Calculate $\lim_{n\to\infty} A^n$.

Q6

There are three rooms, A, B, and C. At every buzzer sound, a cat will either move to another room or stay in a room. Probabilities that the cat in room A will be in rooms A, B, and C after a buzzer sound are 0.4, 0.6, and 0.0, respectively. Likewise, probabilities that the cat in room B will be in rooms A, B, and C after a buzzer sound are 0.2, 0.5, and 0.3, respectively; probabilities that the cat in room C will be in rooms A, B, and C after a buzzer sound are 0.1, 0.7, and 0.2, respectively.

1) What are the probabilities that the cat initially in room A is in rooms A, B, and C, respectively, after the third buzzer sound?

2) What are the probabilities that the cat is in rooms A, B, and C, respectively, after sufficiently many times of the buzzer sound?

Q7

Mechanical parts are stored in four separate boxes. 2000 parts are in Box 1 and 5% of those are faulty. 500 parts are in Box 2 and 40% of those are faulty. Each of the other two boxes contains 1000 parts and 10% of those are faulty.

1) One box is randomly selected from the four boxes and a part is randomly selected from that box. What is the probability that the part is faulty?

2) In the case when the selected part was faulty in 1), what is the probability that the part was selected from Box 2?

3) One part is randomly selected from each of the four boxes. What is the probability that at least one part is faulty?

Q8

There are three containers without a scale. The volumes of the containers are 12 liters, 7 liters, and 5 liters, respectively. Initially, the 12-liter container is filled with water while the 7-liter and 5-liter containers are empty. Explain a procedure to fill one of the containers with exactly 6 liters of water.

Q9

When p is a prime number, $(p-1)!+1$ is exactly divisible by p. Making use of this relationship, find the followings. Note that, for a positive integer m, $m! \equiv m \times (m-1) \times (m-2) \times \cdots \times 2 \times 1$.

1) The remainder of a division of $9!$ by 11

2) The remainder of a division of $58!$ by 61

受験番号 Examinee's number					

東京大学大学院新領域創成科学研究科 海洋技術環境学専攻
令和４（2022）年度大学院入学試験問題（修士課程・博士後期課程）
Department of Ocean Technology, Policy, and Environment
Graduate School of Frontier Sciences, The University of Tokyo
Academic Year 2022 Entrance Examination for Master Course and Doctoral Course

専門科目
「専門基礎科目(論理的思考能力や数理的能力を問う問題)」
Specialized Subjects
"Fundamental Specialized Subject (Problems designed to evaluate logical thinking and basic mathematical skills)"

令和３（2021）年８月 16 日（月）9:30～11:00（90 分）
August 16 (Mon), 2021 9:30～11:00（90min.）

注意事項 Notice

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２．対面受験者はこの表紙にも受験番号を記入しなさい。
Write your examinee number on this test book if you are an on-site examinee.

３．問題は全部で６問あります。６問全てに解答しなさい。
There are 6 questions in total. Answer all of the 6 questions.

４．問題の解答の道筋と答を記入し、答を四角で囲みなさい。
Include your written derivation for your solution and enclose the answer in a box.

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Answer only one question per page. At the top left of each answer sheet, write your examinee's number and the corresponding question number.

３． 解答は指示に従ってアップロードすること。
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第１問

以下の定積分の値を求めよ。

$$\int_0^{\pi} e^x \sin(x) \cos(x)\, dx$$

Q1

Obtain the value of the following definite integral.

$$\int_0^{\pi} e^x \sin(x) \cos(x)\, dx$$

第１問　解答用紙　Q1 Answer sheet

第２問

$O - xyz$ 座標系上に点 A、B、C がそれぞれ（4,5,7）、（2,1,3）、（9,7,6）で与えられるとき、以下を求めよ。

（１） 三角形 OAB の面積

（２） 四面体 OABC の体積

Q2

Find the following, when points A, B and C are given on the $O - xyz$ coordinate system by (4,5,7), (2,1,3) and (9,7,6), respectively.

(1) Area of triangle OAB

(2) Volume of tetrahedron OABC

第２問　解答用紙　Q2 Answer sheet

第３問

観測点 O から 200 メートル離れた同じ水平面上にある地点 P から、鉛直に毎分 25 メートルの速度で風船を上げる。風船の位置を B とし、観測点から風船を見上げた角度を ∠BOP とする。風船が地点 P から 100 メートルの高さに達したとき、∠BOPの時間変化率を求めよ。ただし、風船の大きさは考えない。

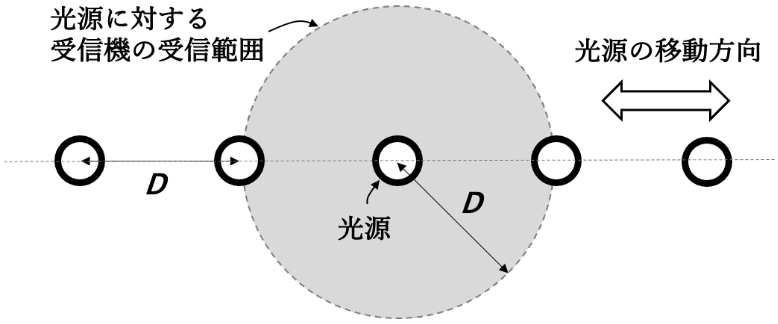
Q3

A balloon rises vertically at the speed of 25 meters per minute from the point P, which is 200 meters away from the observation point O on the same horizontal plane. When the balloon reaches the height of 100 meters from the point P, find the rate of change in time of ∠BOP which is the angle of looking up at the balloon from the observation point O. Assume, B is the balloon position and the size of the balloon is negligible.

第３問　解答用紙　Q3 Answer sheet

第４問

図のように直線上を無数の光源が一定の間隔 D を保って移動している。光は光源から放射状に広がり、受信機は光源から D の距離まで光を受信できるものとする。受信機は光源が並ぶ直線から距離 L だけ離れた平行な直線の上に一定の間隔 $D/2$ で固定されている。光源と受信機の大きさは無視する。

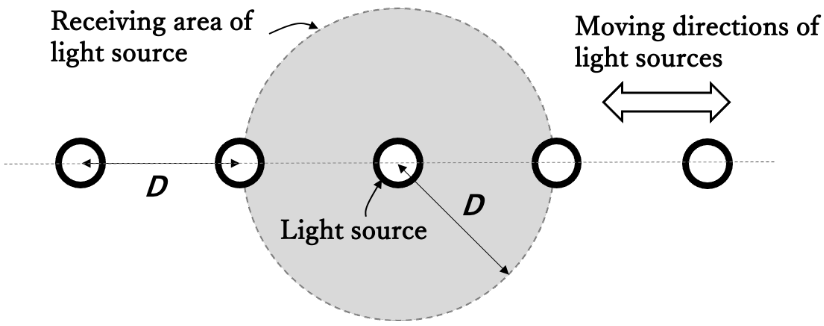


(1) 1 つの光源からの光を常に 1 つ以上の受信機で受信できる L の最大値を求めよ。

(2) 1 つの光源からの光を常に 2 つ以上の受信機で受信できる L の最大値を求めよ。

Q4

A myriad of light sources is arranged with the constant interval D and moves on a straight line as shown in the figure. Light spreads radially from a light source and a receiver can receive light within a distance D from a light source. Receivers are fixed at constant intervals $D/2$ on a straight line which is parallel to the straight line containing the light sources. The distance between the parallel lines is L .



Assume, the size of the light sources and receivers is negligible.

(1) Find the maximum value of L, when the light from one light source can always be received by one or more receivers.

(2) Find the maximum value of L, when the light from one light source can always be received by two or more receivers.

第４問　解答用紙　Q4 Answer sheet

第５問

重力によって落下し、空気抵抗を受ける物体を考える。初速 0 の物体に対し、下記の 3 つの場合において、落下速度の時間変化を求めよ。

(1) 空気抵抗がない場合
(2) 空気抵抗が物体の速さに比例する場合
(3) 空気抵抗が物体の速さの 2 乗に比例する場合

3 つの場合において落下速度のグラフを描き、それぞれを比較しながら考察しなさい。

Q5

Consider a falling object subject to gravity and air drag force. Find the time variations of the object's velocity with an initial velocity of 0 under the following three cases:

(1) There is no drag force.
(2) The drag force is proportional to the object's speed.
(3) The drag force is proportional to the square of the object's speed.

Graph the object's velocity under the three cases. Then compare the curves and discuss their differences.

第５問　解答用紙　Q5 Answer sheet

第６問

投手と打者の間で行うあるゲームにおいて、以下のルールが成立しているとする。

- 投手は打者に向かって勝ち負けが決まるまで球を投げる。
- 投手が投げた球は一定の確率でストライクゾーン内に入る。
- 打者は投げられた球に対してバットを振るか振らないかの選択ができる。
- 打者は投げられた球に対してバットを振ると一定の確率で球を打ち返すことができる。
- 打者はストライクゾーン内に投げられた球を打ち返すとその時点で勝ちとなる。
- 打者はストライクゾーンから外れた球を打ち返しても勝ちとはならない。
- 打者はストライクゾーンから外れた球に対して 4 回バットを振らないとその時点で勝ちとなる。
- 打者は以下の 2 つの事象が合計 3 回起こると負けとなる。
  - a) ストライクゾーン内に投げられた球を打ち返せない。
  - b) ストライクゾーンから外れた球に対してバットを振る。

ある打者はバットを振って球を打ち返す確率が 2 割 5 分である。投手が投げた球がストライクゾーン内に入る確率が 5 割であるとき、以下の問いに答えよ。

(1) この打者が全ての球に対してバットを振らずに勝つ確率を求めよ。

(2) 全ての球に対してバットを振らないのと全ての球に対してバットを振るのとで、この打者が勝つ確率が高いのはどちらか、理由とともに答えよ。

Q6

A game is played between a pitcher and a batter with the following rules:

- The pitcher throws balls towards a batter until the winner is determined.
- Balls thrown enter the strike zone at a certain probability.
- The batter can choose to swing or not to swing at balls thrown.
- When the batter swings the bat, he/she will hit the balls thrown at a certain probability.
- The batter wins if he/she successfully hits a ball thrown inside the strike zone.
- The batter does not win if he/she hits a ball thrown out of the strike zone.
- The batter wins if he/she does not swing the bat at the ball thrown out of the strike zone four times.
- The batter loses if any combination of the following two conditions occurs three times:
  - a) The batter fails to hit a ball that enters the strike zone.
  - b) The batter swings a ball thrown out of the strike zone.

Answer the following questions, when the probability of the batter hitting a ball thrown is 25 % and the probability of the pitcher throwing a ball inside the strike zone is 50 %.

(1) Obtain the probability for the batter to win by not swinging the bat at all balls thrown.

(2) Which has a higher probability for the batter to win: not swinging the bat at all balls thrown or swinging the bat at every ball thrown? Answer with reasons.

第６問　解答用紙　Q6 Answer sheet

受　験　番　号					

東京大学大学院新領域創成科学研究科　海洋技術環境学専攻
令和 5（2023）年度 B 日程大学院入学試験問題（修士課程・博士後期課程）
Department of Ocean Technology, Policy, and Environment
Graduate School of Frontier Sciences, The University of Tokyo
Academic Year 2023 Schedule B Entrance Examination for Master’s Course and Doctoral Course

専門科目

「専門基礎科目（論理的思考能力や数理的能力を問う問題）」

Specialized Subjects

“Fundamental Specialized Subject (Problems designed to evaluate logical thinking and basic mathematical skills)”

令和 5（2023）年 1 月 20 日（金）9:30～11:00（90 分）

January 20 (Fri), 2023 9:30～11:00（90min.）

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第 1 問

関数列$\{f_n(x)\}$が以下の式を満たすとする。ただし、以下の設問では$m,\ n$は自然数、$k_m,\ k_n$は正の定数である。

$$\left.\begin{array}{c} \dfrac{d^2 f_n}{dx^2} + {k_n}^2 f_n(x) = 0, \qquad 0 \le x \le 1 \\ f_n(x) = 0, at x = 0 and x = 1 \end{array}\right\}$$

また、I_{mn}を以下のように定義する。

$$I_{mn} = \int_0^1 f_m(x) f_n(x)\, dx$$

(1) $m \neq n$、$k_m \neq k_n$の場合にI_{mn}を求めよ。

(2) $\{f_n(x)\}$の例を一つ挙げ、$m = n$の場合にI_{mn}を求めよ。

Q 1

Suppose that a sequence of functions $\{f_n(x)\}$ satisfies the following equations where m and n are natural numbers and k_m and k_n are positive constants.

$$\left.\begin{array}{c} \dfrac{d^2 f_n}{dx^2} + {k_n}^2 f_n(x) = 0, \qquad 0 \le x \le 1 \\ f_n(x) = 0,\ \text{at}\ x = 0\ \text{and}\ x = 1 \end{array}\right\}$$

I_{mn} is defined as follows:

$$I_{mn} = \int_0^1 f_m(x) f_n(x)\, dx$$

(1) Obtain I_{mn} when $m \neq n$ and $k_m \neq k_n$.

(2) Give an example of the sequence of function $\{f_n(x)\}$, and obtain I_{mn} when $m = n$.

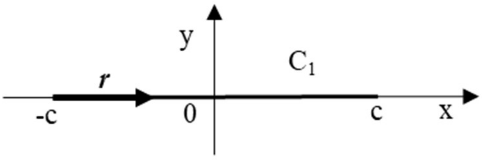
第１問　解答用紙　**Q 1 Answer sheet**

第２問

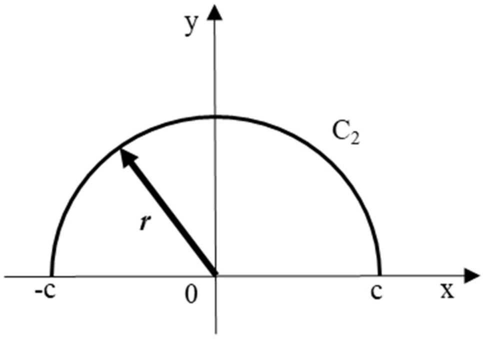
各問に示す積分路に沿って積分Iを求めよ。ただし、$(\boldsymbol{i},\boldsymbol{j})$は直交する単位ベクトルの組、$a$および$b$は任意の定数である。

$$I = \int_{\mathrm{C}} (a\boldsymbol{i} + b\boldsymbol{j}) \cos(ax + by) \cdot d\boldsymbol{r}$$

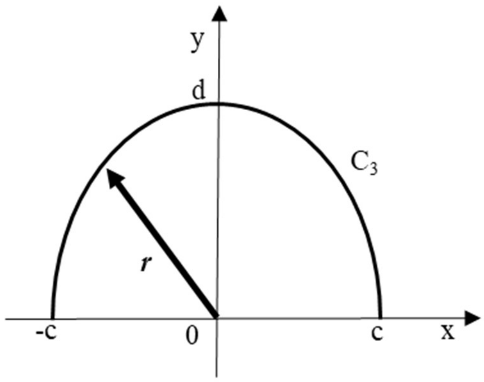
(1) 積分路C_1に沿って積分Iを求めよ。



(2) 積分路C_2に沿って積分Iを求めよ。ここで、C_2は半径cの半円である。



(3) 積分路C_3に沿って積分Iを求めよ。ここで、C_3は短半径c、長半径dの半楕円である。

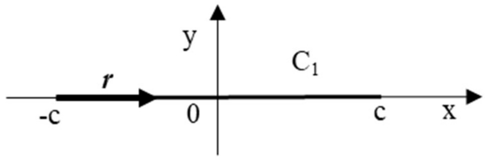


Q 2

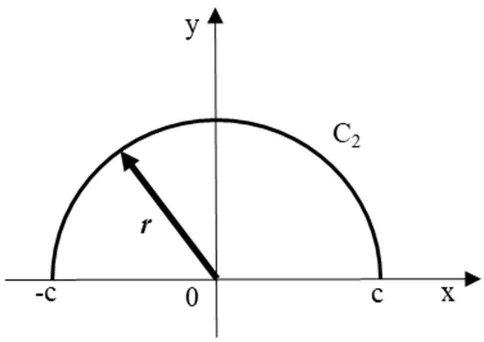
Find the following integral I using paths of integration indicated in each question where $(\boldsymbol{i}, \boldsymbol{j})$ is a set of orthogonal unit vectors, and both a and b are arbitrary constants.

$$I = \int_{C} (a\boldsymbol{i} + b\boldsymbol{j}) \cos(ax + by) \cdot d\boldsymbol{r}$$

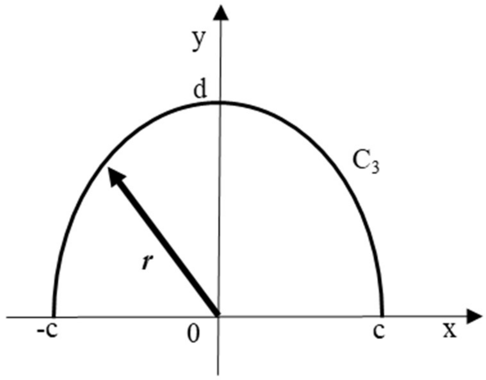
(1) Find the integral I along the path of integration C_1.



(2) Find the integral I along the path of integration C_2 where C_2 is a semi-circle with a radius of c.



(2) Find the integral I along the path of integration C_3 where C_3 is a semi-ellipse with a semi-minor axis of c and a semi-major axis of d.



第 **2** 問　解答用紙　**Q 2 Answer sheet**

第３問

次の行列Aに対しA^k $(k = 1,2,...)$を計算せよ。

(1) $A = \begin{pmatrix} 4 & 8 \\ -2 & -4 \end{pmatrix}$

(2) $A = \begin{pmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}$

Q 3

For each matrix A given below, find A^k $(k = 1,2,...)$.

(1) $A = \begin{pmatrix} 4 & 8 \\ -2 & -4 \end{pmatrix}$

(2) $A = \begin{pmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}$

第 **3** 問　解答用紙　**Q 3 Answer sheet**

第 4 問

次の各問に答えよ。

(1) 半径rの円に内接する正n角形の周の長さLを求めよ。
(2) (1)の結果を用いて、円周率πは 3 より大きいことを証明せよ。

Q 4

Answer the following questions.

(1) Find the length of the circumference L of a regular n-sided polygon inscribed in a circle of radius r.
(2) Using the result of (1), prove that the circular constant π is larger than 3.

第 **4** 問　解答用紙　**Q 4 Answer sheet**

第 5 問

あなたは太郎君とカードゲームをすることになった。各プレイヤーは 4 枚の異なるカードを持っていて、4 枚のカードにはそれぞれスペード、ハート、クローバー、ダイヤのマークが 1 つずつ描いてある。ゲームのルールは以下の通り。

- 1 回のゲームにおいて、プレイヤーは規定の回数のセットを行う。セット毎に、プレイヤーは 4 枚のカードから 1 枚を同時に出し合う。
- 各セットにおける点数は以下のように与えられる。
  - スペードはクローバーとダイヤに勝つ
  - ハートはスペードに勝つ
  - クローバーはハートに勝つ
  - ダイヤはクローバーとハートに勝つ
  - クローバーで勝ったプレイヤーは 2 点を得る
  - スペード、ハート、ダイヤで勝ったプレイヤーは 1 点を得る
  - 勝ち以外は 0 点とする
- 1 回のゲームの間、各プレイヤーの点数は加算される。
- 先に 2 点以上取ったプレイヤーをゲームの勝者とする。
- セットを規定の回数行っても誰も 2 点以上を取れなかった場合、ゲームの勝者はなしとする。

太郎君はセット毎に 4 枚のカードのいずれかを等しい確率で出すものとする。このとき以下の問いに答えよ。

(1) セットの規定回数が 2 回で、あなたは 2 回とも同じカードを出すものとする。このとき、あなたがゲームの勝者となる確率を最大にするにはどのカードを出すべきか。

(2) セットの規定回数が 3 回で、あなたは(1)で答えたカードを 3 回とも出し続けるものとする。このとき、あなたがゲームの勝者となる確率を求めよ。

Q 5

You are going to play a card game with Taro. Each player has 4 different cards, and each card is marked with either a Spade, a Heart, a Club, or a Diamond. The rule is as follows.

- In one game, players play a prescribed number of sets. For every set, players disclose one of their four cards at the same time.
- Each set is scored as follows:
  - Spade beats Club and Diamond.
  - Heart beats Spade.
  - Club beats Heart.
  - Diamond beats Club and Heart.
  - A player winning a set with a Club gets 2 points.
  - A player winning a set with a Spade, a Heart, or a Diamond gets 1 point.
  - Players do not earn points unless they win the set.
- Player's points accumulate during a game.
- The first player to get 2 or more points wins the game.
- If the prescribed number of sets are played and no player gets 2 or more points, there is no winner of the game.

For each set, suppose Taro discloses a card with equal probability between the four cards. Answer the following questions.

(1) Suppose the prescribed number of sets is two and you must disclose the same card in both sets. Which card should you disclose to maximize the probability to win the game?

(2) Suppose the prescribed number of sets is three, find the probability to win the game when you disclose the same card you chose in question (1) in all three sets.

第 **5** 問　解答用紙　**Q 5 Answer sheet**

第 6 問

各面に 1 から 6 までの数字が記載されているサイコロがある。サイコロを振った時に各数字が出る確率は等しいとして、次の確率を求めよ。ただし、N、Mは正の整数であり、$2 < N$、$N/2 < M < N$ の条件を満たすものとする。

(1) N回サイコロを振って、同じ数字が合計でN回出る確率

(2) N回サイコロを振って、同じ数字が合計でM回出る確率

(3) N回サイコロを振って、同じ数字が合計でM回出て、かつM回続けて出る確率

Q 6

Given a standard dice with each of its six faces marked with a different number from one to six. Assuming each of the six numbers shows up with equal probability when the dice is thrown, find the following probabilities. Note that N and M are positive integers and satisfy the conditions: $2 < N$ and $N/2 < M < N$.

The probabilities that

(1) the same number shows up N times in total when the dice is thrown N times.

(2) the same number shows up M times in total when the dice is thrown N times.

(3) the same number shows up M times in total and in M consecutive throws when the dice is thrown N times.

第 **6** 問　解答用紙　 **Q 6 Answer sheet**